Flipped-learning of Introductory Linear Algebra by Utilizing a Free E-Book

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Abstract

This article deals with the authors’ experiences with flipping a basic undergraduate mathematics course in Introductory Linear Algebra, including the platform for flipping a classroom and students’ feedbacks. Contrary to a traditional class where students typically come to the classroom unprepared, in a flipped classroom, students are asked to read content and watch related lectures online before they attend each class session. There are many ways to implement a flipped class effectively. In our experience, we found that a proper textbook and content is necessary in order to expect positive outputs from flipping the classroom. That motivated the writing of a free Linear Algebra e-book that has several interesting new features and our adoption of this e-book for on/offline class sessions of flipped learning. Off-class activities were coordinated and communicated through i-campus, the university’s intranet communication system for teaching and learning. After implementing flipped learning, a mixed reaction from the students was obtained, but students’ ability at problem-solving and their communication skills were generally improved. The main objective of this article is to introduce what we have done during the last 10 years to be prepared to implement our flipped learning environment.

Keywords: flipped learning, flipped classroom, Introductory Linear Algebra, Computer Algebra System, SageMath, i-campus.

1 Introduction

The term ‘flipped classroom’ and ‘flipped learning’ have become popular among university instructors and school teachers who are seeking an innovative approach to their teaching. There is a feature of flipping the classroom in a high profile daily newspaper of The New York Times [Fitzpatrick, 2012], where the author reasons that although the idea of flipped learning is not new, this activity is getting popular thanks to the advancement of technology. Another author argues that flipping the classroom can indeed improve the traditional lecture where for the past hundred of years, teachers put a lot of emphases only on the transfer of information [Berrett, 2012]. Based on his own experience of teaching Physics in Harvard, the pioneer of peer-instruction also argues in the same line of reasoning [Mazur, 2009]. The author provides an evidence that readjusting the focus...
of education from information transfer to helping students assimilate material is indeed paying off.

A formal definition of ‘flipped learning’ is given by the governing board and key leaders of the Flipped Learning Network (FLN). ‘Flipped learning’ is a pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter [Marshall, 2014]. The FLN also released the four pillars or key features of flipped learning, also known as F-L-I-P™. These are ‘flexible environment, learning culture, intentional content and professional educator’. These four pillars include eleven indicators that educators must incorporate into their practice [Hamdan et al., 2013]. Thus, what is a ‘flipped classroom’? The flipped classroom describes a reversal of traditional teaching where students gain first exposure to new material outside of class, usually via reading or lecture videos, and then class time is used to do the harder work of assimilating that knowledge through strategies such as problem-solving, discussion or debates [Brame, 2013].

Although there are several of ways that instructors can implement flipped learning in their classrooms, in principle, the underlying ideas are similar. Students must not receive the teaching material in a passive way. This is one reason why some students do not find the flipped classroom enjoyable. The students collect the information prior to coming to the class. Some activities include, but are not limited to, reading textbooks, watching recorded lectures or listening to podcasts [Berrett, 2012]. There are of course a number of concerns, including the students who feel they are being abandoned to study on their own and a culture shock for students who come from the background where lecturing and rote learning are the norm. Nevertheless, the flipped classroom still has a great potential to create interactive and effective learning activities for all learners [Talbert, 2012]. In fact, we have generated a great deal of new contents from a normal service class in mathematics because students had to say something on/off line class sessions of flipped learning and on-line activities/communication were saved in i-campus.

The term ‘flipped classroom’ is more popular for K-12 education. It was coined and popularized by science teachers Jonathan Bergmann and Aaron Sams back in 2007. They are affiliated with Woodland Park High School in Colorado, and to the best of our knowledge, they are the first who pioneered a strategy to reverse the time frame for lectures and homework [Bergmann and Sams, 2012]. On the other hand, at the college level, the term ‘inverted classroom’ is more commonly used. The term was coined by a group of Economics faculty at Miami University, Ohio in 2000 in a Microeconomics course [Lage et al., 2000]. Although the term might be new, the idea borrows from the centuries old method of giving reading assignments to students outside the class to prepare for an in-class discussion. Thus, in-class time can be devoted more to the group and individual problem solving, critical thinking and experiments.

A comprehensive survey of flipped classroom research has been provided by [Bishop and Verleger, 2013]. The authors reported that student perceptions of the flipped classroom are generally positive. A quasi-experimental quantitative research compares sections of College Algebra using the flipped classroom methods and the traditional lecture/homework structure and its effect on student achievement as measured through common assessments are discussed by [Overmyer, 2014]. The effectiveness of a flipped classroom model on student engagement and achievement as well as the affordances of a flipped model versus that of a traditional model has been explored by [Bormann, 2014]. Research comparing an unflipped class that engages students in some active learning to
a flipped class that creates more time for active learning and to look for measurable differences in student learning, attitude toward course material, and metacognitive skills in Engineering and Mathematics has been described by [Lape et al., 2014].

A design of flipped learning for an undergraduate mathematics course of Linear Algebra at Sungkyunkwan University has been discussed by [Park and Lee, 2016]. Based on the survey that they conducted, the authors found out that the flipped learning does not only increase students’ participation and interest but also improves their communication and self-directed learning skills. A comparison between the traditional lecture format and the flipped classroom model in a Linear Algebra course has also been studied by [Love et al., 2014]. The authors observed that students’ content understanding in the flipped classroom was better than the traditional section and the students’ course perception was also very positive. A study on the effectiveness of flipped learning in Linear Algebra courses has been conducted by [Murphy and Chang, 2014]. The authors found that the students in the flipped classroom have more comprehensive and well-explained responses to the inquiries that required reasoning, creating examples and more complicated mathematical objects. These students also performed superiorly in the overall comprehension of the content material with an increase in the median of final exam score of more than 20%.

Flipped learning has also been implemented in other subjects apart from Linear Algebra. For example, flipped learning in Calculus courses has been implemented for large class sizes and has been discussed by [Jungic et al., 2015]. A study of student performance and perception has been investigated by [Ziegelmeier and Topaz, 2015]. Flipped classroom and instructional technology integration in a college-level information systems spreadsheet course has been discussed by [Davies et al., 2013]. A comprehensive survey on a literature study on flipped learning in STEM subject has been explained by [Dodds, 2015]. The author also provides a prototype flipped classroom within Computer Science major.

This article is organized as follows. Section 2 explains the course logistics, learning management system of i-campus, free electronic textbook and online lecture notes. Section 3 provides features of flipped class in Introductory Linear Algebra course. This includes video recordings, CAS tools of SageMath and some activities related to flipped learning. Section 4 describes the students’ feedback upon the implementation of flipped learning on this course. A reflection from instructors is also discussed. Finally, Section 5 provides conclusion and remark to our findings.

2 Introductory Linear Algebra at Sungkyunkwan University

2.1 Course logistics

In this article, we will discuss our experience implementing flipped classroom for an introductory course in Linear Algebra at Sungkyunkwan University. Linear Algebra (course code GEDB003) is a three-credit course offered as a Basic Science and Mathematics course at this institute. The Department of Mathematics at the Natural Science Campus is offering this course. Typically, around four to five sections of Linear Algebra are offered every semester in both the Natural Science Campus and the Humanities and Social Science Campus of the university. One professor acts as a coordinator of the course and communicates to other instructors on teaching and learning issues including course syl-
labus, material coverage and course assessments, including the midterm and the final examinations.

During the Spring semester of 2016, four sections of Linear Algebra are offered in the Natural Science Campus (NSC) and one section is offered in the Humanities and Social Science Campus (HSCC). Out of these five sections, three sections are designated as flipped classrooms. The sections in the NSC are Classes 42 and 44 of flipped classroom Linear Algebra are taught by one of the coauthors of this paper (NK) and one section (Class 41) is taught by the coordinator of the course and also the coauthor of this paper (SG Lee). Another section (Class 43) is taught by a different instructor (Lois Simon) and is not delivered in the flipped learning format. The maximum number of students in each section is limited to a maximum of 70 students. The majority of the students is majoring in Engineering and thus each class is predominantly male. On a weekly basis, all classes meet twice a week for 75 minutes each. Class 42 meets every Tuesdays at 10:30–11:45 and Thursdays at 09:00–10:15. Class 44 meets every Tuesdays at 13:30–14:45 and Thursdays at 16:30–17:45. Two sections of the course are offered in HSSC and there were handled by one instructor (Sang Woon Yun). The sections of Linear Algebra offered during Spring 2016 is displayed in Table 1.

<table>
<thead>
<tr>
<th>Campus</th>
<th>Section</th>
<th>Instructor</th>
<th>Day</th>
<th>Time</th>
<th>Flipped</th>
<th>Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSSC</td>
<td>01</td>
<td>Sang Woon Yun</td>
<td>Tuesday</td>
<td>12:00–13:15</td>
<td>No</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thursday</td>
<td>15:00–16:15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HSSC</td>
<td>02</td>
<td>Sang Woon Yun</td>
<td>Tuesday</td>
<td>15:00–16:15</td>
<td>No</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thursday</td>
<td>12:00–13:15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NSC</td>
<td>41</td>
<td>SG Lee</td>
<td>Tuesday</td>
<td>09:00–10:15</td>
<td>Yes</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thursday</td>
<td>10:30–11:45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NSC</td>
<td>42</td>
<td>NK</td>
<td>Tuesday</td>
<td>10:30–11:45</td>
<td>Yes</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thursday</td>
<td>09:00–10:15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NSC</td>
<td>43</td>
<td>Lois Simon</td>
<td>Tuesday</td>
<td>12:00–13:15</td>
<td>No</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thursday</td>
<td>15:00–16:15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NSC</td>
<td>44</td>
<td>NK</td>
<td>Tuesday</td>
<td>13:30–14:45</td>
<td>Yes</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Thursday</td>
<td>16:30–17:45</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Schedule details of sections of Linear Algebra courses offered in both NSC and HSSC during the Spring semester 2016.

### 2.2 Learning management system of *i-campus*

The communication between the instructor and the students is accommodated through an online learning management system known as *i-campus*. *I-campus* is an internal platform of learning management system for students and faculty alike at Sungkyunkwan University. The system is operated by our university has features similar to more general learning management systems, including Moodle, Blackboard Learn and MyEdu. The screen shots of *i-campus* before and after the login are shown in Figure 1. *I-campus* is specifically designed for online learning and supports blended learning lecture, cyber lecture and specialized telecturing. With a thorough preparation on teaching materials, instructors have the liberty to upload relevant course materials and reference materials, to initiate open discussion, to assign homework and tasks and to the monitor students’ progress continually.
In i-campus, the instructor can view his or her assigned classes for a particular semester and view the students’ enrollment for each assigned course. I-campus also has a feature for assigning teams for the students, either manually or automatically and either randomly or based on students’ name or students’ ID. The course syllabus, however, can only be viewed but cannot be modified. To modify the course syllabus, one must enter another system known as Advanced Sungkyunkwan Information Square (ASIS). The recording of every class session can be found in i-campus under the section of ‘Lecture Support’—‘Manage Lecture Materials’. These recordings are arranged on a weekly basis. The instructor can also manage and distribute assignments, initiate discussions, administer exams, conduct surveys and receive and respond to personal notes. The screen shots of the i-campus page for faculty before and after the login are shown in Figure 2.

Figure 1: Screenshots of the SKKU’s i-campus homepage, before login (left panel) and after login (right panel).

Figure 2: Screenshots of the SKKU’s i-campus page for faculty, instructors and managers, before login (left panel) and after login (right panel). On the right panel, we can see some announcements posted by the instructor and the teaching assistant to the students under the ‘Communication’ part and ‘Manage Notices’.
2.3 Free e-book

A new model of interactive digital mathematics textbook has been introduced by [Lee et al., 2017]. The mathematics courses include Calculus, Linear Algebra, Differential Equations, Probability and Statistics, and Engineering Mathematics. The authors in particular discussed a digital textbook on Linear Algebra and focus on their experience in using digital contents and interactive laboratories for developing a new model for a digital textbook in Linear Algebra. The authors also include some appraisal from the readers who feel that the digital textbook is very useful for them, a staff employed by the Bank of Korea and a faculty member from University of California at Berkeley in the US.

We adopted a textbook developed by our own faculty and the electronic version of this book is freely available from the web address [http://ibook.skku.edu/Viewer/LA-Text-Eng](http://ibook.skku.edu/Viewer/LA-Text-Eng) and the Portable Document Format (PDF) file can be downloaded from the above address. It is accessible to anyone and the students can download and use it for free of charge and thus we are promoting the utilization of electronic book (e-book). The students who wish to obtain a hard copy version of the textbook can order it from [http://pod.kyobobook.co.kr](http://pod.kyobobook.co.kr) by ‘print on demand (POD)’ service from the largest bookstore in South Korea for an affordable price of around US $15 at the current exchange rate.

The English version of this textbook is based on and is translated from the original version of Korean language. The Korean language version of the e-book is also freely available and accessible online from [http://ibook.skku.edu/Viewer/LA-Texbook](http://ibook.skku.edu/Viewer/LA-Texbook). The front cover of both versions is shown in Figure 3. This Linear Algebra e-book contains ten chapters, and the title of each chapter is given in the ascending order as follows: Vectors, System of Linear Equations, Matrix and matrix algebra, Determinant, Matrix Models, Linear Transformations, Dimension and Subspaces, Diagonalization, General Vector Spaces and Jordan Canonical Form. The appendix of the e-book contains sample exam questions with their solutions and some worked out exercises. These exercises have been solved, revised and finalized by the students and the final check is conducted by the main author of the e-book.

The textbook contains many worked examples that will enhance students’ understanding and in learning Linear Algebra. It also contains embedded interactive materials with the computer algebra system of SageMath. Every section of the book includes links to recorded video lectures on YouTube and practice web pages. SageMath codes are included in the textbook and thus the students can practice using a computer or a mobile device. They can simply copy and paste the commands and the computer code from the e-book to a SageMath worksheet or SageMath cell server and modify them accordingly to enhance their understanding of the material. For instance, by playing with a larger size of a matrix or by changing some figures from simple ones to more complicated figures, students are able to perceive the usefulness of the computer algebra system and the aid that it provides.

2.4 Online lecture notes

In addition to the Linear Algebra textbook in digital format, recently one of us has also developed online lecture notes. These notes are accessible at [http://matrix.skku.ac.kr/LA/](http://matrix.skku.ac.kr/LA/) As can be seen in this website, the lecture notes are arranged according to each chapter of the e-book, and there are also ten chapters in total. Whenever possible, each example is accompanied by SageMath code where there is a link to the corresponding web page. This is very useful for class demonstration using SageMath as the instructor.
can simply press the ‘Evaluate’ button and the computational result will be displayed accordingly. This action saves time in comparison to copy and paste the computational codes to a separate cell server.

Furthermore, the YouTube video links of the recorded in-class lectures are also displayed below the URL links of the lecture notes. These video links are arranged according to the section number and related topic systematically. This allows the students to access and to watch the recorded video lectures on the relevant topic before they come to the class. Hence, providing the links which are accessible to the students supports the structure of flipped classroom where the students can gain basic information outside the classroom by listening to podcasts or watching recorded video lectures. In-class time can thus be dedicated to more rigorous problem-solving activities, discussion and feedback. Past sample exams including both midterm and final examinations are also available on the web page.

3 Flipped class in Introductory Linear Algebra

3.1 Video recordings

Flipped learning is one feature of active learning. The students are expected to be prepared when they attend and come to the class. There are a number of ways to encourage the students to do this, including reading assignments and watching recorded video lectures. While the students tend to disregard the former one, the latter one has a more favorable preference among the students. Of course, embedding an assignment element into the recording also enforces the students to watch the video before every class session. There are several options when it comes to choosing what videos should be assigned to the students to watch. For instance, Khan Academy has more than 2400 free videos in many different subjects. See https://www.khanacademy.org/

For this Introductory Linear Algebra course, as instructors, we have decided to create our own video recordings. Depending on the materials, the duration of the recorded videos varies from ten minutes to a full-lecture of around 50 minutes. All recorded video lectures are accessible via YouTube. The students watch some parts of the entire recordings before they attend class sessions. Screen shots of the screencast video recording are shown Figures 4 and 5.
Figure 4: (Left) A screen shot from a YouTube page which shows a screencast video material of Linear Algebra on determinant, eigenvalue and eigenvector. (Right) A screen shot of the SKKU’s Sage cell server with an example of calculating a determinant of a $3 \times 3$ matrix and the inverse of a matrix.

The left panel of Figure 4 shows a screen shot from a YouTube page showing a screencast video material of Linear Algebra on determinant, eigenvalue and eigenvector. The right panel of Figure 4 shows a screen shot of the SKKU’s SageMath cell server with an example of calculating a determinant of a $3 \times 3$ matrix and the inverse of a matrix. The left panel of Figure 5 shows a screen shot from a YouTube page which shows a screencast video material of Linear Algebra on subspace. The right panel of Figure 5 displays a screen shot of the Analytics data page from YouTube. From the Analytical data provided by YouTube, we observe that the most popular videos that the students watch are the topics on Subspace, Linear Transformation and Finding the Jordan Canonical Form, where each video has been viewed for more than 600 times.

Figure 5: (Left) A screen shot from a YouTube page which shows a screencast video material of Linear Algebra on subspace. (Right) A screen shot of the Analytics page from YouTube.

### 3.2 Computer Algebra System SageMath

SageMath is a computer algebra system (CAS); a free open-source mathematics software. As can be found in its official website, the mission of dissemination SageMath is to create a viable free open source alternative to Magma, Maple, Mathematica and Matlab.
See [http://www.sagemath.org/](http://www.sagemath.org/) It is distributed under the terms of the GNU General Public License version 2+. It can be utilized to assist computations in Linear Algebra, Combinatorics, Numerical Methods, Number Theory and Calculus.

In this course of Introductory Linear Algebra, SageMath is utilized both for in-class activities as well for out-of-class activities. Some in-class activities include demonstrations of solving Linear Algebra problems using SageMath. For this, the instructor can simply go to an online ‘single cell’ version of SageMath accessible at [https://sagecell.sagemath.org/](https://sagecell.sagemath.org/). The cloud computational platform is available from our own university’s cell server and is accessible at [sage.skku.edu](http://sage.skku.edu). A screen shot of this cell server is shown on the right panel of Figure 4. Some alternative SageMath cell servers available in South Korea amongst others are provided by Korea National Open University (KNOU), which is accessible at [http://mathlab.knou.ac.kr:8080/](http://mathlab.knou.ac.kr:8080/) and by the National Institute for Mathematical Sciences (NIMS), which can be accessed at [https://sage.nims.re.kr/](https://sage.nims.re.kr/).

Out-of-class activities include assignments for students to solve problems using the software. As mentioned previously in Subsections 2.3 and 2.4, the free electronic textbook and the online lecture notes on Introductory Linear Algebra contain many worked examples on solving problems using SageMath. Some examples of in-class and out-of-class activities will be discussed in the following subsection.

### 3.3 Flipped class activities

The class time is dedicated to problem-solving session and workshop on the use of technology. The way an instructor teaches the class can be done by demonstrating some computational examples using SageMath. Both the students and the instructor may attempt the computational examples with small size matrices and simple problems from Linear Algebra that can be easily checked by pencil and paper.

Furthermore, the students do their homework assignments with a larger size of matrices, and more general vectors and Linear Algebra concepts. Naturally, this raises a question whether it is possible to have higher level discussions on Linear Algebra concepts. This is where the instructor plays a role in assisting out the students both inside and outside the classroom. The instructors stimulate discussions among the students by posing problems and challenge the students to come up with the best and the nicest solutions. A collection of these problems and solutions is compiled in a folder and the folder will be shared before the exams. The students can also ask problems before they come into the class, participate in the discussion, receive answers and feedback to their inquiries and improve their understanding.

The following is some examples of in-class and out-of-class activities utilizing CAS SageMath. In-class activities include problem solving using pen and paper as well as using CAS. For pen and paper, the students are often asked to find determinant of $2 \times 2$ or $3 \times 3$ matrices. For some special cases, a larger size of matrix can be calculated by hand, but SageMath is powerful to find a determinant for any size of matrix. Examples include calculating the determinants of the following $3 \times 3$ and $5 \times 5$ matrices

\[
A = \begin{bmatrix}
1 & 5 & 0 \\
2 & 4 & -1 \\
0 & -2 & 0
\end{bmatrix}
\quad
B = \begin{bmatrix}
3 & -7 & 8 & 9 & -6 \\
0 & 2 & -5 & 7 & 3 \\
0 & 0 & 1 & 5 & 0 \\
0 & 0 & 2 & 4 & -1 \\
0 & 0 & 0 & -2 & 0
\end{bmatrix}.
\]

The fastest way to calculate the determinant of matrix $A$ is by using the cofactor expansion across the third row. To calculate the determinant of matrix $B$, one must recognize that...
the final three rows and three columns of matrix $B$ contains matrix $A$. Hence, one can calculate the determinant of matrix $B$ by using the cofactor expansion along the first column, and then expanding along the second column and after that multiplying with the determinant of matrix $A$. Even though the students have calculated the determinant of matrix $A$ before, some of them fails to utilize this result when calculating the determinant of matrix $B$. Using SageMath, the computation of these determinants only takes a few seconds. The following is the command that can be inputted into the SageCell server.

\[
A = \begin{pmatrix}
1 & 5 & 0 \\
2 & 4 & -1 \\
0 & -2 & 0
\end{pmatrix}
\]

print "det(A) =", det(A)

\[
B = \begin{pmatrix}
3 & -7 & 8 & 9 & -6 \\
0 & 2 & -5 & 7 & 3 \\
0 & 0 & 1 & 5 & 0 \\
0 & 0 & 2 & 4 & -1 \\
0 & 0 & 0 & -2 & 0
\end{pmatrix}
\]

print "det(B) =", det(B)

One example of out-of-class activities is an assignment for the students to use SageMath or any other CAS to find a dominant eigenvalue and the dominant eigenvector of a $2 \times 2$ matrix $A$ with a given initial condition $x_0$, where

\[
A = \begin{pmatrix}
-7 & -12 \\
8 & 13
\end{pmatrix} \quad x_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.
\]

The students needed to solve this problem numerically by implementing the ‘power method’ algorithm and they were required to provide a minimum of ten number of iterations. A dominant eigenvalue of a matrix is the largest eigenvalue in the absolute value sense and the dominant eigenvector is the eigenvector corresponding to the dominant value. An example similar to this problem can be found in the free e-book, including the SageMath code and the corresponding computational result. A common mistake is that the students calculate eigenvalues and the corresponding eigenvectors of the matrix analytically using SageMath and other CASs. Another common mistake that was observed is that although the students attempt to use power method, but they did not provide a minimum ten number of iterations. The following is a SageMath code to find the dominant eigenvalue and the corresponding eigenvector for the example above.

```python
from numpy import argmax, argmin
A = matrix([[-7,-12],[8,13]])
x0 = vector([1.0,0.0]) # Initial guess of eigenvector
maxit = 20 # Maximum number of iterates
dig = 8 # number of decimal places to be shown is dig-1
tol = 1e-7 # Tolerance limit for difference of two consecutive eigenvectors
err = 1 # Initialization of tolerance
i = 0
while (i<=maxit and err>=tol):
    y0 = A*x0
    ymod = y0.apply_map(abs)
    imax = argmax(ymod)
    c1 = y0[imax]
    x1 = y0/c1
    err = norm(x0-x1)
    i = i+1
    x0 = x1
print "Iteration Number:", i-1
print "y"+str(i-1)+" =", y0.n(digits=dig),
    "c"+str(i-1)+" =", c1.n(digits=dig),
    "x"+str(i)+" =", x0.n(digits=dig)
print
```
The computational result is displayed as follows.

Iteration Number: 0
y0 = (-7.0000000, 8.0000000) c0 = 8.0000000 x1 = (-0.87500000, 1.0000000)

Iteration Number: 1
y1 = (-5.8750000, 6.0000000) c1 = 6.0000000 x2 = (-0.97916667, 1.0000000)

Iteration Number: 2
y2 = (-5.1458333, 5.1666667) c2 = 5.1666667 x3 = (-0.99596774, 1.0000000)

Iteration Number: 3
y3 = (-5.0282258, 5.0322581) c3 = 5.0322581 x4 = (-0.99919872, 1.0000000)

Iteration Number: 4
y4 = (-5.0056090, 5.0064103) c4 = 5.0064103 x5 = (-0.99983995, 1.0000000)

Iteration Number: 5
y5 = (-5.0011204, 5.0012804) c5 = 5.0012804 x6 = (-0.99996800, 1.0000000)

Iteration Number: 6
y6 = (-5.0002240, 5.0002560) c6 = 5.0002560 x7 = (-0.99999360, 1.0000000)

Iteration Number: 7
y7 = (-5.0000448, 5.0000512) c7 = 5.0000512 x8 = (-0.99999872, 1.0000000)

Iteration Number: 8
y8 = (-5.0000090, 5.0000102) c8 = 5.0000102 x9 = (-0.99999974, 1.0000000)

Iteration Number: 9
y9 = (-5.0000018, 5.0000020) c9 = 5.0000020 x10 = (-0.99999995, 1.0000000)

Iteration Number: 10
y10 = (-5.0000004, 5.0000004) c10 = 5.0000004 x11 = (-0.99999999, 1.0000000)

From this result, we observe that the dominant eigenvalue is 5 and the eigenvector corresponding to the dominant eigenvalue is \((-1, 1)\).

4 Students’ Feedback and Reflection

4.1 Students’ feedback

There are various reactions from the students upon the implementation of flipped class. The feedback is collected before and after the midterm test period, around Week 7 and Week 9, respectively. There exists another feedback collected at the end of the semester too, around Week 15 and Week 16. The following is some feedback in the connection to flipped class.

“I hate flipped class .... I want a traditional class.”
“No flipped class, please. Lecture in the classroom is more useful.”
“Flipped class is not good. I feel that flipped class does not have a vitality.”
From these comments, we observe that some students do not favor flipped class but prefer a class with the traditional teaching style instead. This is understandable as they can only sit down and listening in the traditional teaching style class, while in the flipped class, they are required to do preparation and participate actively during the class, and thus more effort is required from the students’ side.

Regarding the video recordings, the following is some comments that the students write.

"It is hard to understand what the video says because it is too short, so I have to study alone. It feels that I don’t take any class but just to studying alone."

“The videos are just reading presentation slide. Please add more explanation.”

“...it was not good that the lecture was given in the video. In the video, the professor just reading the presentation slide.”

“I think just studying with recorded video is hard, so your teaching during the class is needful.”

“I need more ‘detailed explanation’ in the recorded video. I could not understand the content fully unless I watch another video from YouTube, etc.”

“Video lectures are not helpful. So, it is hard to understand the concept of Linear Algebra.”

From these comments, we discover that many things need to be improved when it comes to producing recorded video lectures. In a course like Introductory to Linear Algebra, we need to inform the students of many important definitions, terminologies, lemmas, propositions and theorems and we thought that informing this information through the video will be a good idea. Therefore, we attempt to read these definitions and theorems so that we do not need to repeat telling the same thing during the class time. However, students interpreted this action differently. They informed us that simply reading the texts is not a good thing to do. It may inform them, but does not necessarily they understand the presented material. Therefore, a more detailed explanation is needed and some examples accompanying the presentation will do good.

4.2 Reflection

Implementing a flipped class certainly is not an easy task. The students need to be introduced clearly what to be expected from them at the beginning of the semester and throughout the semester repetitively. We need to convince the students that their efforts are worthed when they were asked to spend more time for class preparation outside the regular class contact hours. All of us are fully aware that our students are busy too with other classes and extracurricular activities. Therefore, it is not always easy for them to do preparation, even just merely watching the videos, on a regular basis for the entire period of the semester.

On a technical side, we realized that teaching using video can be pretty hard and producing high quality videos is not an easy task either. We did not receive any official training on how to produce good quality teaching video recordings. It takes several times of trials and errors. We also need to ensure that the duration of the videos is not too short and not too long either. For many of the students, particularly those who are used to with traditional style of teaching, receiving information for the first time through reading a textbook or watching the video recordings might have difficulty in grasping the presented materials, particularly if English is not their first language. We also observed that providing examples in our videos can help the students to make connections between Linear Algebra concepts and how to use these concepts in problem solving.
5 Conclusion and Remark

In this article, we have discussed our experience of flipping the classroom in an Introductory Linear Algebra class at Sungkyunkwan University. A new feature of our teaching includes developing a new, free electronic textbook that can be accessible to everyone in the world and disseminating online lecture notes on a single web page that can be used by both students and instructors alike. We have embedded the use of technology in our teaching, in particular exploring the power of open access and free CAS SageMath for computational purposes. We have also created recorded video lectures, either in a screen-cast format or in the form of real live classroom teaching. These videos are accessible to the students from the YouTube platform and they can watch primarily not only before they come to the class sessions, but also after the class is over if they wish to review the lectures again at their own pace. While the students gather basic information on the topics of Linear Algebra outside the class by listening to podcasts or watching recorded video lectures, in-class activities can be geared toward problem-solving, discussions and providing feedback. Furthermore, we utilize the intranet online learning management system of i-campus for teaching and learning as well as communication outside the classroom. This platform is used to provide additional teaching materials, including lecture notes and other supplementary information, to send announcements to the students, such as the information on the upcoming quizzes, to distribute assignments and to collect the students’ submissions and to discuss by using ‘Question and Answer’ platforms where the students can post questions and the instructor, the teaching assistant and other students can respond accordingly.

A collected survey on the students’ opinion to the implementation of flipped learning indicates a mixed reaction. Some students like and enjoy the idea of flipped classroom, where they can watch the recorded video recording before the class and participate actively during the problem-solving session in the class. Other students find the video recording are less interesting to watch because it just reading through the definitions and theorems. The feedback that the students provide is very useful for the improvement of flipped classroom for our future classes. Overall, it is a great experience for the instructors and the students to participate in flipped learning combined with many elements of technology.

References


